Brother John Taylor Fellowship

A Place for Mathematical Structure in the Classroom

A series of papers by Mark Gronow

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The Brother John Taylor Fellowship was established in 2014 to encourage academic study by people dedicated to the principles of Catholic schooling.

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Part 1 – A place for mathematical structure in the classroom

Mathematical structure and structural thinking in mathematics:
What are they and why they matter?

This article is the first of three that have been written as a fulfillment of the Br John Taylor fellowship for educational research. The author's research on this topic coincided with the completion of a Master of Research at Macquarie University, under the supervision of Professor Joanne Mulligan. The research undertaken in mathematics education looks at how to improve student engagement in mathematics through attention, to what mathematical researchers have termed "mathematical structure".

This, the first article reports on the current research on mathematical structure and structural thinking, and how it impacts on student engagement in mathematics. The second article describes mathematical structure within the context of the current NSW K-10 mathematics syllabus, and explains the importance of developing students' structural thinking in mathematics. The third and final article associates mathematical structure with the research on mindsets, in particular mathematical mindsets and how current practices have created fixed mindsets in the teaching and learning of mathematics, and how attention to mathematical structure increases students' structural thinking, develops growth mindsets to support deeper understanding of mathematical procedures, and concepts to improve engagement.

Student disengagement in mathematics

It is well recognised that an increasing number of students are choosing General Mathematics for the NSW Higher School Certificate. This was highlighted publicly after the 2015 HSC results were released (Ting, 2015). The cause of students not attempting higher levels of mathematics and disengaging from the subject lies in earlier years before senior school calculus courses are considered.

Students begin disengaging from mathematics from early years of primary school, and this continues into junior secondary school years. In a longitudinal case study investigating student engagement in middle years of schooling, Attard (2013) collected data through interviews, focus group studies, and classroom observations and showed that positive pedagogical relationships between teacher and students were important as the foundation for students maintaining engagement in mathematical learning. In a previous study, Attard (2010) found that teachers without a pedagogical background in mathematics had difficulty explaining mathematical concepts to the students. With the decline in mathematics teacher graduates, the number of teachers without mathematical pedagogical experiences has increased, which has added to student disengagement. Motivational attitudes toward mathematics in early secondary school was also identified by Plenty and Heubeck (2011), they stated that students' decisions about their perceived mathematical ability impacted on their engagement with the subject.

Enagement in mathematics is linked to how it is taught. Skemp (1976) produced his seminal paper about instrumental versus relational understanding in learning of mathematics. Skemp emphasised the need to transform mathematics teaching from an instrumental, or procedural methods, to a relational, or conceptual understanding. Kilpatrick, Swafford, & Findell (2001), and Watson & Sullivan (2008) described procedures as the ability to recall mathematical facts readily. This describes how most people remember of their mathematical experiences: rote learning facts, and procedures to be reproduced in timed tests. Sullivan (2011) stated that Skemp's theory of relational understanding is aligned to conceptual understanding as an appreciation of mathematical ideas and relationships.

Teaching of procedures in mathematics is inherent in the teaching and learning of mathematics, as it presents an approach to complete a problem. The procedure represents the memorised method used to solve a problem, the concept being the mathematical theory, model, or idea the student needs to understand. The danger in the over use of a procedural understanding approach is that a reliance on memorising procedures without understanding the concept, or why a particular procedure is used leaves the student with a lack of understanding why a procedure is used, leaving he students unable to think deeply about the mathematical concepts presented (Mason, Stephens, and Watson 2009; Boaler, 2015b). Further to the over reliance of procedural understanding is demonstrated by research from Prescott and Cavanagh (2006) and Bobis (2000) who identified how new teachers adopted a procedural approach early in their teaching career.

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Procedural understanding, as previously mentioned, can be characterised by memorising a method. For example, to find the area of a rectangle, students are taught to multiply length by breadth, where the length and breadth are given as two separate numbers. Students, in a procedural manner, simply learn to multiply the two numbers given without understanding what area is, and why the two numbers are multiplied. This approach will yield the correct answer for the area of a rectangle, but is meaningless if the student does not recognise the length and breadth as being the adjacent sides of the rectangle. The words become pointless as many other words or symbols can be used. The multiplication process works for a rectangle but the formula cannot be applied to other plane shapes, for example a triangle. A procedural understanding requires completion of a number of similar examples, explaining the steps to get the answer, setting the students a number of similar examples to practise by repeating the process, and finally assessing the ability to repeat the process in a timed test. Procedural understanding is important in mathematics, but not as the focus. Memorising a method does not develop a deep understanding of concepts. Procedural understanding is specific to the examples given, but the process is unlikely to transfer to other situations.

Conceptual understanding requires knowledge of the basic principles of mathematics. It encourages the learners to think about the mathematics they are learning rather than recalling facts and processes. It is flexible and can be generalised to new situations.

Australian mathematics teachers have been identified as teaching principally towards a procedural understanding. In the Third International Mathematics and Science Study (TIMSS) 1999 video study, Australia was shown to have a higher proportion of nonqualified mathematics teachers, and teaching methods were dominated by a procedural approach (Lokan, McRae, & Hollingsworth, 2003). the study showed that all mathematics teachers – whether qualified or unqualified - tended towards this approach. There is a need for all teachers to be aware of the negative effects of a purely procedural approach has on learning of mathematics. The TIMSS video study identified those countries with the highest TIMSS scores tended towards more conceptual understanding in mathematics teaching.

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What is mathematical structure?

Mathematical structure as focus of a report by Mason, Stephens & Watson (2009) states that teachers' awareness of mathematical structure improves students' engagement in mathematics. If this is the case, then there is a need to identify how the classroom teacher can do this.

As a definition of mathematical structure, Mason et al. (2009) stated it as being "the identification of general properties which are instantiated in particular situations as relationships between elements or subsets of elements of a set" (p. 10). They believed that appreciating structure is powerful in developing students' understanding of mathematics and that attention to structure should be an essential part of mathematical teaching and learning. Mathematical structure is a precursor to structural thinking, which can be associated to cognitive structures, producing schemas that are essential in mathematical thinking and successful learning. Mason et al. (2009) stated that mathematical structure is not taught. Rather, it is an understanding of how the procedures and concepts are connected to support student learning.

In light of a significant amount of research into mathematical structure, Taylor and Wade (1965) proposed a theoretical definition as the formation and arrangement of a mathematical system within mathematical properties. Others have also referenced mathematical structure. Jones and Bush (1996) use a "building blocks" metaphor to describe mathematical structure, stating that mathematical structure is like the foundation of a building on which the content is built. They identified structural thinking in mathematics as a vehicle for helping students understand and answer the "why" questions in mathematics. In a different approach to mathematical structure, Schmidt, Houang, and Cogan (2002) were concerned with the deeper sense of mathematical structure as it connects content to deeper mathematical understanding. More recently, Mulligan and Mitchelmore (2009) identified structural thinking in preschool patterning strategies.

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Mathematical structure and structural thinking in mathematics

In aligning mathematical structure to current teaching methods, Mason et al. (2009) identified mathematical structure as closely aligned to conceptual, and procedural understanding. Understanding why a procedure is used and the concept being developed helps the student to develop structural thinking awareness. Mason et al. believed that students stop learning when mathematical structure is not appreciated in the classroom. Teachers need to initiate students into mathematical structure, and cultivate it in order to mature this appreciation. By presenting research that supports this view, Mason et al. (2009) argue that students who are not encouraged to observe mathematical structure in their mathematics learning, or are not engaged in structural thinking processes, become blocked from thinking deeply about mathematics. Their point is that to develop structural thinking skills in students, teachers must have an awareness of what is mathematical structure.

In furthering their argument Mason et al. (2009) connected mastering procedures, and understanding concepts to structural thinking. They stated, that the learner would understand the relevance of the mathematics being taught, rather than relying on memorising, when the teacher's focus is on mathematical structure. Effective mathematical thinking involves being able to use, explain, and connect mathematical properties. They use specific examples of how mathematical structure bridges the gap between procedural and conceptual understanding of mathematics in teaching and learning.

Attention to mathematical structure, as presented by Mason et al. (2009) as the overarching theory that complements and supports, procedural and conceptual understanding of mathematics should be addressed in every mathematics classroom. They provide evidence that students' mathematical understanding is enhanced when mathematical structure is the focus of learning. To achieve this, teachers need to acknowledge mathematical structure in the content taught, the pedagogy employed, and they need to avoid relying on procedural understanding in teaching mathematics.

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Mathematical structure and pedagogical content knowledge

Teacher's understanding of mathematical structure is a significant component of pedagogical content knowledge (PCK), described by Shulman (1987) as an understanding of what to teach as well as how to teach it, is a requirement for good teaching of mathematics.

The mathematical content taught requires an awareness of mathematical structure by the teacher for effective communication to the learners. Mathematical structure enables the teacher to explain the content, so students can relate to it. The ability to demonstrate structural relationships is essential in the mathematics teacher's pedagogy. Attention has been given to developing teacher pedagogical content knowledge (PCK) as a means of improving student learning by mathematical education researchers (Bobis, Anderson, Martin, & Way, 2011; Hill & Ball, 2004). As a component of mathematical pedagogy, mathematical structure should be included as an important part of PCK. Clarke, Clarke, and Sullivan (2012) recognised the importance of mathematical content knowledge and Vale, McAndrew, and Krishnan (2011) found that understanding both mathematical procedures and concepts improved with an awareness of mathematical structure. The knowledge of the content is essential before the concepts and procedures can be taught. Mathematical structure links these concepts, or as Mason et al. (2009) describes connects the "mythical chasm" between procedures and concepts.

Bobis (2000) did not use single out mathematical structure in her article, but identified components of mathematical structure that effective mathematics teachers understand: the interconnectedness of ideas, the ability to select, use efficient and effective strategies, challenge students to think, and encourage them to explain, listen, and solve problems. Bobis identified mathematical structure through the strategies that develop structural thinking in the students. Deeper understanding of mathematical structure will encourage mathematics teacher to use these strategies in the classroom that encourage structural thinking.

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Mathematical structure and student engagement in mathematics

Teachers of mathematics can address the issue of engagement by paying greater attention to mathematical structure in their classroom. Teachers who have an awareness of mathematical structure can develop structural thinking that engages students. (Mason et al., 2009; Mulligan & Mitchelmore, 2009; Taylor & Wade, 1965).

In their study about improving participation rates in mathematics, Brown, Brown, and Bibby (2008) surveyed over 1,500 students in 17 schools. Results from a questionnaire found that the perceived level of difficulty of the mathematics and personal lack of confidence were reasons for students not continuing with mathematics. These factors, along with a dislike and boredom, as well as a perceived lack of relevance, were also related to students' decision not to study mathematics at senior secondary school level.

Mathematical structure aims to increase student engagement. The ability to think structurally, Mason et al (2009) asserted, gives students intrinsic reward from their enjoyment in mathematics. It is not about the mark on a test or being the fastest to answer the question. Further, Mason et al. (2009) concluded that a teacher's awareness of structural relationships would transform students' mathematical thinking and their disposition to engage.

Teachers understanding of mathematical structure

Various mathematics education researchers have proposed individual definitions of mathematical structure that have similarities to a broad concept, but display individual distinctions (Barnard, 1996; Jones & Bush, 1996). Others have attempted to identify how mathematical structure and structural thinking impact on students' mathematical understanding (Jones & Bush, 1996; Mason et al., 2009; Mulligan & Mitchelmore, 2009; Vale et al., 2009). Despite this large body of research about mathematical structure, there is a lack of research about teachers' understanding of mathematical structure and how they teach with reference to mathematical structure in junior secondary schools.

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To improve engagement in mathematics, students must become confident learners, to do this, all mathematics teachers, qualified and nonqualified need to consider mathematical structure in their pedagogical practice. The necessity for teachers to put this at the forefront of their teaching does not require a change from their current teaching habits. It simply requires a greater awareness of how students learn mathematics, and how they think when learning mathematics. Through mathematical structure, students develop structural thinking skills, which will develop a greater sense of connection to the mathematical content being taught. Once connection is made then an understanding of the procedures used and the mathematical concepts being developed will support learning. Students no longer need to rely upon memorising facts and formulae, which, are of little benefit for long-term mathematical development.

Current pedagogical practices have failed our students. The challenge for teachers, school executive, and parents is to put aside traditional methods and change the mindset that teaching mathematics for memory to complete timed tests, where a pass viewed as a determinant of those who can and those who can't. The message this sends to young student creates a mindset of not good enough. Through the work on mindsets of Carol Dweck (2006) and Jo Boaler (2015b), as will be discussed in further detail of part 3 of this report, we now know that all people are capable of mathematical thinking. Boaler (2015a) describes the importance of making mistakes and how the brain activity when mistakes are made and corrected. Memorising does not improve brain development. Boaler describes how mathematical thinking can be achieved, and under the right conditions students' confidence, as mathematical learners, will increase. Teachers' awareness of mathematical structure, the development of structural thinking, and an attention to these mindsets will be the first and important step in this process.

Conclusion

The overemphasis of procedural methods in the mathematics classroom has left students disengaged in mathematics. Deeper thinking of mathematics is being achieved with a richer conceptual understanding of the content. This can be achieved by teachers' developing an awareness of mathematical structure so students can understand the procedures and concepts and be able to think structurally about the mathematics, develop greater insight and appreciation in the mathematics to be learnt.

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This is the second of three reports written to complete the Br John Taylor fellowship requirements. The theme of these reports is that of mathematical structure, this report combines research in this field, and where it applies to the current NSW K-10 mathematics syllabus through working mathematically. The author completed this research as a requirement for the Master of Education degree at Macquarie University under the supervision of Professor Joanne Mulligan.

Focus on Mathematical structure in teaching and learning

Mathematical structure is not a term commonly used by teachers in a mathematics classroom environment, but it does have a long history of use by mathematics education researchers. The notion of mathematical structure was identified by Mason, Stephens and Watson (2009) as existing as far back as Euclid, but it might be assumed that mathematical structure has been a part of mathematics since mankind started to think mathematically.

While not identifying with the term mathematical structure, Skemp (1976) made a distinction between what he referred to as instrumental and relational understandings of mathematical learning. He explained that instrumental understanding was learning a number of fixed plans with starting points and finishing points and explanations of what to do along the way, whereas relational understanding involved building up a conceptual structure or a schema that offered an unlimited number of starting points toward any finishing point, with multiple paths to get there. These terms are identified within the psychology of mathematics education literature through the work of Fischbein and Muzicant (2002, p. 248), and they represent different approaches about how mathematics is taught. The delineation between procedural and conceptual learning and understanding is bridged by mathematical structure, so that deeper thinking about the mathematics being taught is achieved. Mason et al. (2009) maintains that mathematical structure covers understanding of both procedures and concepts.

Starting from a premise, that the mastering a procedure is important when taking advantage of learning opportunities to make mathematical sense, Mason et al. (2009) stated that it is of little value to the learner if it remains a procedure. Procedural learning simply places a burden on the learner to remember, but when procedures are associated with some appreciation of mathematical structure the learning shifts from memorising to understanding the concepts. The negative effects of procedural learning, identified by Richland, Sigler, and Holyoak (2012), are that it leaves the learner ineffective at any mathematical reasoning. It is structural thinking that allows learners to have confidence in manipulating the procedures taught, and apply the concepts to mathematical problems. Using the expression "conceptual structure", Richland et al. (2012) acknowledged that this process allowed learners to make predictions regarding how procedures relate to the solutions, and develop new understandings about the concepts. In their study, Richland et al. (2012) examined mathematics knowledge of students who had completed the K–12 mathematics sequence and found these students were unlikely to have flexible reasoning in mathematics. Students in this study saw mathematics as a collection of procedures, rules, and facts to be remembered, and found that this became increasingly difficult as they progressed through the curriculum.

Mathematical structure foundations are in the connections between procedural understanding, which is the doing of a problem, and conceptual understanding, which is described as the knowing or understanding of why a particular procedure is used. In their article, Richland et al. (2012) proposed that students' long-term ability to transfer and engage in mathematical knowledge is achieved through mathematics instruction that focuses on making connections between the mathematics learnt as procedure or concept.

Teachers' awareness of mathematical structure

Research about mathematics teachers' awareness of structure is related about pedagogical approaches and teacher professional learning. In a study conducted in Australia, Vale, McAndrew, and Krishnan (2011) examined out-of-field (i.e. nonqualified) mathematics teachers, after they completed a professional development course for junior secondary mathematics teachers on mathematics syllabus content and pedagogy. The researchers explored teachers' understanding of mathematical connections and their appreciation of mathematical structure. An appreciation and awareness of mathematical structure was identified through teachers' recognition of mathematical relationships and properties. These teachers were then able to make connections between these relationships and properties, which they implemented in their teaching to promote student structural thinking.

The study by Vale et al. (2011) demonstrates that when teachers were taking the position of the learner, their appreciation of how structure makes connections with more complex concepts. A benefit that teachers experience when taking the role of the learner is to understand how students' think when working on mathematical problems. This experience helps when creating pedagogical practices that support the development of structural thinking.

The teacher when aware of mathematical structure can identify connections between previous, current, and future learning of procedures and concepts. The expression "knowledge at the mathematical horizon" (Vale et al., 2011, p. 169) is used to describe how teachers' mathematical knowledge is required to enhance students' future mathematical learning. A definition of mathematical structure as "building blocks" (Jones & Bush, 1996) would adequately describe this idea. All mathematics learnt forms the foundation for future mathematics to be learnt.

An important outcome from the Vale et al. (2011) research was the impact that the teachers involved experienced. They felt that their deepened awareness of mathematical structure, and their ability to explore structure had increased their desire to develop a deeper understanding of pedagogical knowledge for classroom practice. In addition to this, the teachers understood that an emphasis on procedural understanding limits students in their understanding of mathematics.

Mathematical structure in mathematics curricula

Mathematical structure can be found in current international mathematics curriculum documents. The American mathematics curriculum document *Common Core State Standards for Mathematics Initiative* "*Common Core*" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) includes "look for and make use of structure" as one of the eight standards of mathematical practices used to demonstrate what students are doing when they learn mathematics. It recognises mathematical structure in student learning and understanding of mathematics.

Mathematical structure can be identified in the Australian Curriculum—Mathematics (Australian Curriculum, Assessment and Reporting Authority, 2015) through the four proficiency strands of understanding, fluency, problem-solving, and reasoning. These proficiency strands reflect the multidimensional aspects of mathematical structure, and support how the content is taught through the thinking, and doing of mathematics. Essentially, these proficiency strands can be aligned with the development of structural thinking skills. In the Australian curriculum, the lack of use of the term does not mean that the concept of mathematical structure is not.

In the NSW mathematics syllabus for the Australian curriculum (NSW Board of Studies, 2012), the proficiency strands of the Australian curriculum are re-worked as working mathematically. Mathematical structure can be identified in working mathematically through the communicating, problem solving, reasoning, understanding, and fluency components. Working mathematically can be associated with students' behaviour and affective learning patterns. In the NSW Assessment, Certification and Examination manual (Board of Studies NSW, 2016) values and attitudes exist as the affective learning or emotive aspect when doing mathematics.

Teachers' use of mathematical structure revealed in working mathematically

In a survey, 39 mathematics teachers were interviewed by Cavanagh (2006) to examine the extent they used working mathematically in their teaching. Although this study was concerned with the NSW mathematics syllabus before the introduction of the Australian curriculum, the results are relevant. A small number of teachers interviewed were able to describe what working mathematically involved and applied it to their teaching. The majority of teachers had a very limited understanding of what working mathematically meant. As working mathematically can be aligned to mathematical structure, it appears that teachers' lack of working mathematically awareness could transfer to a similar lack of awareness of mathematical structure.

The teachers Cavanagh (2006) interviewed identified time pressure as one reason for not applying working mathematically in their teaching. They said that the content-driven curriculum did not encourage teachers to focus on these components of mathematical learning, and the need to prepare students for examinations was a barrier to incorporating activities that encouraged working mathematically. These are realistic and conspicuous reasons for mathematics teachers not to practice working mathematically in their day-to-day teaching. Similarly, teachers might not practice pedagogical approaches espousing mathematical structure because of these conditions.

Mathematical structure when identified in the working mathematically component of the NSW K-10 Mathematics (BOSTES, 2016) curriculum, complements the spirit of the NSW mathematics K-10 syllabus. It aims to promote an effective mathematical pedagogy that promotes students' mathematical thinking.

Professional development to increase awareness of mathematical structure

Teachers' awareness of mathematical structure is important as mathematical structure is a determinant of students' mathematical understanding. Mathematical problems used by teachers during classroom instruction need to reflect their awareness, and appreciation of mathematical structure. Teachers need to transfer their mathematical structure awareness, as suggested by Mason et al. (2009), to their students. Students will then develop structural thinking through an awareness of mathematical relationships, and properties.

The effectiveness of mathematical structure when focusing on mathematical content and pedagogy was identified in a professional development program created by Vale et al. (2011). Teachers involved in this program reflected on their learnt experiences of understanding mathematical concepts, and appreciation of pedagogical knowledge. Their reflections indicated that they had deepened, and broadened their knowledge of teaching junior secondary mathematics, as well as developing their capacity to support students' learning of mathematics. The researchers indicated that there is a need for further research in the area of teachers' awareness of mathematical structure and how teachers could be encouraged to embed structure in their teaching practice. The development of professional development programs such as that implemented by Vale et al. (2011) can cultivate teachers' awareness of mathematical structure, so they can implement strategies to deepen students' mathematical understanding through structural thinking.

The need to identify how teachers can embed structure, through working mathematically, into to their lessons then becomes a focus of mathematical pedagogy. The teacher's awareness of mathematical structure, through mathematical structure, becomes a critical before it can be persuasively applied into the mathematics lesson.

Mathematical structure and assessment practices

A focus on mathematical structure means a focus on students' understanding mathematical procedures and concepts through structural thinking. Understanding is the intrinsic rewards that motivates students, as Mason et al (2009) acknowledged, the knowing how, and why, motivates students to engage in mathematical learning, not the mark that appears on an assessment task. Boaler (2015) goes further to say that rigid assessment practices are detrimental to students learning. Timed tests usually require students to memorise facts and procedures to be reproduced in a controlled environment, and these can be in the form of quick quizzes of 5-10 minutes in the classroom, longer 30 – 60 minute end of topic tests, assessment tests, and 1.5–3 hour examinations that mimic the NSW Higher School Certificate (HSC). The expectations of students, parents, teachers, school executive, and the community in general is that by giving students smaller tests situations then they are slowly being prepared for the longer and more important examinations. This certainly is the case for student preparation to complete the HSC, but the growing number of students disengaging in mathematics by the manner that students are being prepared for this examination is proving to be a critical issue as to why this one of the reasons why this approach is failing to engage students, and motivating them to undertake higher levels of mathematics.

Granted that students need to be prepared for examinations at the higher levels of mathematics, but the over use of procedural teaching processes that junior secondary students are exposed to recall the content required has created what Mason et al.

(2009) called "memory burden". Young students, simply cannot retain the amount of information required, they do not process it all in timed test situations. Attard (2010) identified that the transition from primary to secondary school was identified as causing anxiety in young students. Students are already anxious about mathematics and doing tests just supports this further. Nervousness interferes with ability to perform well in tests. There are many other issues that are not considered as influencing the students' performance. Yet, it is the result on these timed assessments that students complete in junior secondary school determine their personal attitude, motivation, and confidence towards the subject. The result has seen fewer students enjoying mathematics learning, declining engagement in mathematics in the first years of secondary school leads to fewer students attempting higher levels of mathematics in senior years

When the teacher focuses on mathematical structure through the working mathematical component of the NSW mathematics syllabus, teachers are aware of all aspects of students mathematical thinking, and in particular the affective, or emotional, learning component. How a student feels about their ability, or their self-efficacy towards the subject, will impact on their motivation and interest in learning. Teaching towards procedures and memorisation of facts, places all mathematical learning into one category and fails to encourage students to think mathematically. When there is more emphasis on structural thinking students are able to think deeply about the mathematics learnt, and to apply their understanding learning test situations, with greater depth of understanding.

Mathematics teachers need to put mathematical structure through, the working mathematically aspect of the NSW mathematics curriculum, rather than a purely content focus, at the forefront of all their pedagogical practices in the classroom. By developing teaching strategies through these components, students will develop a deeper understanding of mathematics, increase their structural thinking skills, remember, and apply the mathematics to other situations beyond the timed test. Alternative teaching strategies, and assessment approaches that encourage students to think mathematically, communicate their ideas, listen to others, make alternative decisions and mistakes, and not be afraid to be wrong are components of structural thinking that should be encouraged in all mathematical learning. Mathematics that has one procedure, and one answer does not extend students learning and thinking. It simply restricts students thinking that there is only a right or wrong way, and if you know the right way then you are "good at maths" there is no alternative if the path chosen is the wrong way.

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This is the final of three reports written to complete the Br John Taylor fellowship requirements. The theme of these reports is that of mathematical structure, this report focuses on how mathematical structure challenges current educational assessment and reporting procedures, and how it links with current research on mindsets. The author completed this research as a requirement for the Master of Education degree at Macquarie University under the supervision of Professor Joanne Mulligan.

Focus on mathematical structure

Mathematical structure is an alternative to the memorising of facts and procedures approach in mathematics teaching. Mathematics teachers can implement mathematical structure into their teaching through a greater awareness of how, and what students' think when learning mathematics. The important perspectives of affective, and cognitive learning need to be considered. The affective learning domain, which is the emotional component of learning, influences the student's cognitive understanding of the mathematical concepts, and his or her ability to reproduce these as learnt procedures. Mathematical structure is not content to be taught, it is implicit in the teaching and learning of mathematics to support students' understanding, which includes student well-being. Feeling good about learning mathematics will improve engagement, motivation, and success. This discussion presented in this article identifies how the awareness of mathematical structure, through the working mathematically component of the NSW mathematics syllabus, will benefit students in the mathematics classroom, and post secondary school. The concept of growth mindsets is introduced to support mathematical structure as a crucial element for effective change to current pedagogical procedures in mathematics to improve student learning and engagement.

Mathematical structure and working mathematically are linked through familiar components of the working mathematically outcomes in the NSW K-10 mathematics syllabus (NSW Board of Studies, 2012). Working mathematically outcomes taught in isolation to the content, they are embedded in the content strands of syllabus: number and algebra, measurement and geometry, and statistics and probability like mathematical structure, they are the required for developing deeper understanding of mathematical concepts and procedures, and are essential in supporting students' affective learning of mathematics. Unfortunately these outcomes, and their potential to effect student engagement, are not fully realised. If they were, we would not be seeing the decline of student engagement in mathematics, and enrolment in advanced mathematics courses. In schools, mathematics programmes, scope and sequences, and units of work recognise working mathematically outcomes for administrative purposes, such as: assessment, reporting, and school inspections. Yet, are rarely acknowledged in the day-to-day teaching, or effectively addressed in assessment tasks. Cavanagh (2006) demonstrated this in teachers' lack of awareness, and inclusion, of working mathematically in their every day teaching.

Timed tests and homework

The focus on timed test assessments, and procedural understanding has created students who are: disinterested, disengaged, unsuccessful and disliking mathematics (Boaler, 2015a). Many parents, and some mathematics teachers, will argue that the importance of a mark, the class/year average on a test, and a rank position in the group to show how the student is achieving in mathematics relative to others. The popular belief is that this is how mathematics has always been assessed, and should remain so, as everyone understands, and accepts it.

Conversely, against popular opinion, this is not the appropriate way to assess student mathematical understanding and ability. Additionally, it is not effective in developing engaged, motivated, and successful learners, as will be discussed further. Teachers set tests as a means to satisfying the rigid assessment procedures imposed by school policy. Tests allow teachers to answer parents' questions about students' achievement, work ethic, and participation in class. As well, tests promote a progression through the heavy content burden of the mathematics syllabus.

The traditional procedural approach to teaching mathematics to prepare students for tests and examinations is criticised by mathematical researchers, such as Boaler (2015a, 2015b) as failing our students. Students in their first year of secondary school are subject to unrealistic measurement of their mathematical ability in their performance on timed test. Tests do not measure mathematical ability, they are more like to test: effectiveness of home study, ability to manage time to complete all of the test, reading and comprehension skills to interpret the test questions, and especially, ability to overcome the anxiety under pressure. Attard (2010) identified that students transitioning between primary and secondary school are alienated from mathematics because of these rigid assessment structures. Boaler (2015b) questions why we use these harsh assessment techniques when assessing student mathematical ability, as there is no other situation in adult work life that requires this type of performance assessment. Why then is it necessary to impose this regime upon our youngest, and most vulnerable? In most cases, the answer is because it is expected, timed tests separates those who can, from those who cannot, to reward those who can. This is has not been successful in developing a society where mathematical skills and interests are common, and success at mathematics a higher level levels are encouraged and rewarded.

As part of a procedural approach to teaching, students in junior secondary schools are expected to spend time in class, and at home of completing repetitive tasks that focus on memorising, rather than understanding, a procedure, in preparation for tests. The procedure has little importance in young peoples' world, and is irrelevant beyond the content being taught at that time. Classwork followed by repeated tedious and meaningless homework does not develop motivated, and interested learners. Conner, Pope and Galloway (2009), found that homework created stress for the students, meant a loss of family time at home, and had little impact on achievement. There are few young students that are good at this procedural learning, particularly when their technological world allows them to achieve interesting, and relevant information quickly. Students see mathematics, in this light, as irrelevant, and boring (Brown, Brown, & Bibby, 2008).

Twenty-first century learners

Conrad Wolfram creator of wolfram alpha in his Ted talk https://www.ted.com/talks/conrad_wolfram_teaching_kids_real_math_with_computers (Wolfram, 2010), talks about the importance of computers in learning mathematics. He is aware that computation skills are regarded as essential components of learning mathematics, but in today's technological world they are not as important as they were 20 years ago. Computers, and calculators perform complex mathematical tasks more efficiently that humans can. Wolfram says that the skills needed to be developed in the mathematics classroom are not computational skills, but the ability to pose a question, to transfer knowledge from the real world to a mathematical model, and to relay the model back to the real world to see if the original question is answered.

Boaler (2015a) noted that in 1970 the most valued employment skill was writing and computation, whereas in 1999, teamwork and problem solving are the top two employable skills, with computational skills ranked 12th. There is no need to teach students in 2015, the same skills that were expected in the 1970s. Today's generation of learners' are required to be structural thinkers who can work mathematically, with have the ability to problem solve, think critically, analyse, communicate, and reason. These are the essential skills of the future. Today's students become bored very quickly with repetitive, and boring tasks that are represented by procedural learning. They need to be excited and engaged in mathematics to become successful learners of the future.

Mathematical mindsets

The creation of those who can, and those who cannot do mathematics is encouraged, and continually promoted through a procedural focus of memorising required for completing timed tests. To reinforce success on these tests, schools will rank students, further organising students into those who can, and cannot, then tell them how good or bad at mathematics they are by their placement in a ability level classes. Boeler, Wiliam, and Brown (2001) pointed out the harmful messages students receive when placed into ability level classes, both at the top and bottom.

There is a belief is that higher achieving students will be better grouped together in one class, and those less successful in their own class. Paek and Foster (2012) found that for many students that achieve good results on tests discontinue advanced mathematics courses when graded into higher level mathematics classes, as they have not developed a conceptual understanding as their successful experiences are as a result of memorising, through procedural learning. Within, the highest streamed classes there are students who feel inadequate, as they are not achieving at the same level as the better students. Marsh et al. (2014) researched the negative effect of putting capable students into selective schools, they found that this lowers personal self esteem, confidence and creates feelings of inadequacy because when compared to others they feel they are never good enough, the result is a lower achievement compared to true potential.

Parents reinforce poor mathematics attitudes by telling their children that “I was not good at maths either”, as if it was a family trait. Eccles and Jacobs (1986) found that when mothers said to their daughters “I was not good at math at school” their daughter’s results declined. There is a belief that some people are gifted with a maths gene, or that not all people are capable of achieving at mathematics. This view has been contradicted by the work of Carol Dweck, a professor in psychology at Stanford University http://mindsetonline.com/abouttheauthor/. Dweck (2006) introduced the concept of mindsets, and has transformed the way we view intelligence. Intelligence is not fixed at birth, making some people born smarter than others. Dweck says that we can achieve our potential through the way we approach the situation. She describes people with growth mindsets, as those who believe that smartness increases with hard work, whereas people with fixed mindsets believe that while you can learn things, you cannot change your smartness. When people change their mindsets they start to believe they can learn at higher levels and change their learning pathways. In this Tedtalk video, Dweck (2014), https://www.youtube.com/watch?v=hiiEeMN7vbQ, describes fixed and growth mindsets. Mindsets have been supported through foundations in neuroscience, and how brain plasticity allows for the growth of new neuron pathways. Boaler (2015b, p. 13) describes how brain activity increases when mistakes are made and neuron pathways are formed through mistakes, not by remembering correct responses. She asserts that there is a need to encourage students to think about the mathematics they do, and be prepared to make mistakes. Students need to be exhilarated in learning from the mistakes made, not penalised, as is done in tests. She is critical upon the procedural understanding approach to teaching mathematics as it only allows for correct answers, and discourages mistake making.

Jo Boaler is a professor in mathematics education at Stanford University. Having taught mathematics in the UK and USA, from junior secondary to undergraduate levels, her focus is on improving mathematics education for all. She has researched, and written extensively on how students’ learn mathematics effectively, and holds a strong belief that mathematics is accessible to everyone. Her website www.youcubed.org is followed worldwide by teachers, parents and students. Her books: *What's math got to do with it?* (Boaler, 2015a), and *Mathematical mindsets* Boaler (2015b) are changing the way we think about teaching mathematics.

Boaler (2015b) does not describe mathematical structure in the same manner as Mason, Stephens, and Watson (2009). Her focus is towards effective learning, and that by creating a learning environment that has this focus, then the intrinsic reward students need to engage in mathematical learning is their excitement in mathematical understanding. This approach supports Mason et al. (2009) view that intrinsic rewards of understanding mathematics are achieved through structural thinking.

Mathematics has been taught as a competition, often based on speed responses, that only a few students are acknowledged as capable of. Boaler (2015b) expresses the danger of schools, and some teachers, to identify these students as gifted in mathematics, and ignoring those who work hard to achieve. She points to research that shows that these students while doing well at school often do not perform, as well in adulthood. Where as, students who are persistent, work to overcome difficulties, and challenge themselves to do their best despite setbacks, are the ones who go on to achieve later in life. To perform well in year 7 and 8, students need to be good at timed tests, these students are good at memorising, but do not necessarily have the conceptual understanding that is required to do well in more advanced mathematics. Boaler (2015a) identifies these students, often high achieving girls, as having fixed mindsets, and lack the confidence to attempt more difficult mathematic for fear of failure. Where as the students with growth mindsets are not afraid of making mistakes, and learning from their mistakes. Students with the growth mindset will achieve at a greater level in year 11 and 12. Students who relied on memory to achieve do not achieve at the higher levels, as their methods of learning do not develop structural thinking skills. Students who worked towards understanding the concepts improved, and continue to improve, as they focused on conceptual understanding.

Boaler has created an online learning course for teachers, parents and students through her www.youcubed.org website. This video https://www.youtube.com/watch?v=2jixf8gYdT0 (Jo Boaler, 2014a) is from this online learning course for teachers. It explains mindsets in a mathematical classroom, and gives a practical example of how a middle school teacher took the problem of 'one divided by a third', to create a learning environment where students were open and engaged in a discussion, and applied structural thinking to solve the problem. To solve the problem were able to view alternative pathways, use visual displays to explain how they got their solution, and discuss why some methods were incorrect. Students were involved in structural thinking, and the teacher's attention to mathematical structure on understanding of the division process, deepened the teacher's pedagogical understanding of listening to the students explain their thinking, and developed a pedagogical process beyond a procedure. An algorithm to solve the problem, as would be taught in a procedural based classroom was acknowledged, but students were asked to 'make sense' of the question before the algorithm was introduced. The algorithm would still be used to solve other problems, but students understanding of division with fractions allows them to think deeply about the question being asked, and apply the knowledge learnt to the new situation. Students' ability to transfer knowledge, make connections between previously learnt matter and new work, and the relationships between content are essential components of structural thinking. The teacher's ability to structure a lesson that allows students to develop these structural thinking skills is essential for higher mathematical thinking.

Including mathematical structure

Mathematics teachers are not required to make radical changes to their current pedagogical practices to include mathematical structure into their teaching. Mason et al. (2009) states that mathematical structure cannot be taught, and it is the teachers' awareness of this concept that develops students' structural thinking. Vale et al. (2011) demonstrated how mathematical structure improved teachers understanding of mathematics, as well, made them aware of how students learn mathematics.

Mathematics teachers are aware of the need to teach concepts, and that students need to spend more time in consolidation of these concepts, and develop greater awareness of mathematics that makes sense to them. Boaler (2015a) calls it 'making sense', and once students make sense of the mathematics they are learning, then they develop a growth mindset that encourages engagement in mathematics. Once understanding of mathematical concepts occurs, then engagement will follow.

In the NSW K-10 mathematics syllabus we see the theme of working mathematically presented before the content strands. Addressing mathematical structure as an awareness of working mathematically through communicating, problem solving, reasoning, understanding, and fluency are easily associated with Boaler's (2015b) work. In her book, *Mathematical Mindsets* (Boaler, 2015b,) presents the working mathematically components: reasoning, problem solving, and communication as essential requirements to mathematical learning.

Working mathematically becomes a focus of what teachers teach, not as an additional component to be included at the end of the content, or an additional question in an assessment task to satisfy school registration and accreditation requirements. Teachers need to teach towards mathematical structure in every lesson. Mathematical structure through working mathematically can be in the instructions given, the activities assigned, the discussion and collaboration between students.

What needs to happen?

The responsibility of improving student engagement in mathematics is not only the mathematics teacher's responsibility. School executives must be forward thinking, and plan towards giving teachers the support they need to make the changes required to develop competent, and engaged learners of mathematics. The school community must be educated that mathematics ability is not about tests scores, class averages, and ranks. It is about independent and creative learners willing make mistakes, and feel confident to be challenged by their mistakes. Teachers need to be given the freedom to work their class at pace that is supportive of individual learner, common tests that suggest 'a one size fits all' is not benefiting the higher ability students, and simply demoralizes the students who need more time, and support.

Students, and teachers must be educated in mindsets, and encouraged to dismiss teaching dominantly to procedural understanding, which has proven unsuccessful, and does not develop a growth mindset to support challenging concepts, particularly for girls.

The current situation

Our current focus on achieving band 6 scores in the HSC does not encourage students to take the higher levels of mathematics, able students see that a good result in General Mathematics as leading towards a higher ATAR than a mediocre results in the higher mathematics course (Pitt, 2015). Rylands and Coady (2009) found this was detrimental to performance at university, as courses studied at school, and not university entrance scores determined success at university. Students who attempt the higher mathematics course do better at university. Their conceptual understanding of mathematics, and their structural thinking skills are at a higher level, this creates a mindset that encourages the challenge presented by difficult concepts. Whereas, the General Mathematics course relies on procedural understanding, without the development of deeper thinking skills.

As a result of the high stakes of HSC performance, schools are trending students towards General Mathematics to market their school with higher numbers of Band 6 results. A band 5 in Mathematics is substantially better than a band 6 in General Mathematics, but does not get recorded in newspapers or school newsletters. Schools should be encouraging students to work towards the more challenging courses, and be promoting the increasing number of students attempting such courses. Teachers actively encourage students to take lower level mathematics because it will make their class results look better when HSC analysis are required, and the school executive scrutinises student results against teacher performance.

Capable students are being encouraged to do less challenging mathematics courses to improve the schools marketability within the community. Schools need to resist these publicity ploys so students can develop to their full potential. Teachers should not feel threatened by students doing higher level of mathematics, they should be rewarded for encouraging, and supporting students who want to commit to the subject to meet the challenges of more difficult mathematical concepts. The process that is currently in place, is the continual dumbing down of students' ability and rewarding of success that requires less effort.

The mathematics learner we want to produce

A future where young people will grow as adults willing to be forward thinkers, risk takers, collaborators, and problem solvers with growth mindsets capable of continually learning, and accepting any challenge placed before them, will be the measure of a school's success. Schools should not look at the current Year 12 HSC results, but look at where its students are five to ten years after they have left school. The HSC does not always identify successful adults; if schools marketing featured student success later in life we would see a marked difference in how we are educating our younger students. The need to focus on test scores would not be the priority, developing capable, and competent thinkers with growth mindsets would be the goal. In mathematics, to achieve this, the teachers need to introduce mathematical structure into their lessons through he working mathematically components of the NSW mathematics syllabus. School executives need to encourage teachers to pursue this direction through professional development and administrative changes to school assessment and reporting practices. Finally the school community needs to be informed that the goal of mathematics is not a mark on the page, but being engaged to want to work harder, challenge oneself, and make mistakes.

The following link from Jo Boaler (2014b), youcubed.org website, sums up the arguments presented here, www.youcubed.org/think-it-up.

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